# Finger Exercises 4

*Vectors & the Dot Product*

≈ 60 minutes, plus a 10-minute bonus

CS 453 — Computer Graphics

**What this is:** Short drills that lock in Lecture 4 — vectors as arrows, length and direction, the dot product and its shadow reading, projection, and bouncing. Nothing needs more than 5 minutes, except the starred bonus at the end.

**How to do it:** On **paper** first, no computer. Draw a little grid whenever a question has coordinates in it — the picture catches sign mistakes that the arithmetic hides. If an item takes longer than 5 minutes, skip it, finish the rest, come back. A2 drops Friday and is built entirely on these ideas.

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## Part A • Do You Remember?

RECALL ≈ 5 min

**A1.** A vector is an *instruction*: $(4, -1)$ says "__________ right, __________ down". It remembers its ___________ and its ___________, but *not* where it starts.

**A2.** A point is a ___________ ("here"); a vector is a ___________ ("that way"). The trip from point $P$ to point $Q$ is __________ − __________ — *destination minus start.*

**A3.** The length of $(x, y)$ is $|\mathbf{v}| =$ __________________. A *unit* vector has length exactly ______; to make one, divide a vector by __________________.

**A4.** The dot product $(a_x, a_y) \cdot (b_x, b_y) =$ ____________________. Two vectors go in; a ___________ comes out.

**A5.** Sign reading: dot $> 0$ means _______________; dot $< 0$ means _______________; dot $= 0$ means _______________.

**A6.** Shadow reading: with $\hat{\mathbf{b}}$ one step long, $\mathbf{a} \cdot \hat{\mathbf{b}}$ is the length of ________________________________.

## Part B • Arrows on the Grid

COMPUTE ≈ 5 min

Draw each one on a small grid before you write the numbers.

**B1.** Point $P = (1, 4)$, point $Q = (6, 7)$. The trip $Q - P$ is ( ______, ______ ), read aloud as "_________________________".

**B2.** Which of these arrows are the *same* vector as $Q - P$ above? Tick all that apply.

☐ from $(0, 0)$ to $(5, 3)$ ☐ from $(2, 2)$ to $(7, 5)$

☐ from $(5, 3)$ to $(0, 0)$ ☐ from $(3, 1)$ to $(8, 3)$

**B3.** Walk $(2, 5)$, then walk $(4, -1)$. Where do you end up relative to where you started? $(2, 5) + (4, -1) =$ ( ______, ______ )

**B4.** Scaling — fill in: $3 \cdot (2, -1) =$ ( _____, _____ ) $-(2, 5) =$ ( _____, _____ ) $0.5 \cdot (8, 6) =$ ( _____, _____ )

**B5.** Classify each expression as a **point**, a **vector**, or **nonsense**. (`pos` and `target` are points; `vel` and `gravity` are vectors.)

`target - pos`: __________ `pos + vel`: __________ `vel + gravity`: __________ `pos + target`: __________

## Part C • Nudge the Ball

TRACE ≈ 5 min

The ball sits at $(1, 2)$; the target sits at $(9, 6)$. Each nudge moves the ball a **quarter** of the way to the target.

**C1.** First: how far right, how far up? `dx` = ______, `dy` = ______.

**C2.** Nudge 1: the ball moves ( ______, ______ ) and lands at ( ______, ______ ).

**C3.** Nudge 2 — recompute from the *new* spot: `dx` = ______, `dy` = ______; the ball moves ( ______, ______ ) and lands at ( ______, ______ ).

**C4.** Compare the two moves. Is the ball speeding up or slowing down? Will it ever *arrive* exactly? What one change gives it a steady speed instead?

## Part D • Length and Direction

COMPUTE ≈ 6 min

**D1.** Lengths — all three come out whole: $|(6, 8)| =$ ______ $|(5, 12)| =$ ______ $|(-8, 15)| =$ ______

**D2.** The unit vector of $(6, 8)$ is ( ______, ______ ). Check it: its length is $\sqrt{\_\_\_\_\_\_ + \_\_\_\_\_\_} =$ _____.

**D3.** Which is longer, $(7, 1)$ or $(5, 5)$? Decide *without* a square root.

**D4.** Steady speed. Position $(2, 2)$, target $(8, 10)$, speed 5 per nudge. The trip is ( _____, _____ ), its length is _____, the one-step direction is ( _____, _____ ), so one nudge moves ( _____, _____ ) to ( _____, _____ ). How many nudges to arrive?

**D5.** What goes wrong if the target is *exactly* where the ball already is? What should the direction be, and why is the test `length == 0.0` the wrong guard?

## Part E • Read the Sign

COMPUTE ≈ 7 min

**E1.** Compute each dot product and tick the reading.

| pair | dot | same way | perpendicular | opposite |
|---|---|---|---|---|
| $(2,5)\cdot(3,-1)$ | ______ | ☐ | ☐ | ☐ |
| $(1,2)\cdot(-4,2)$ | ______ | ☐ | ☐ | ☐ |
| $(-3,1)\cdot(2,-5)$ | ______ | ☐ | ☐ | ☐ |
| $(5,0)\cdot(0,-7)$ | ______ | ☐ | ☐ | ☐ |

**E2.** A guard stands at $(4,4)$ and looks *up*: $\mathbf{f}=(0,1)$. For each player position, find $\mathbf{rel}$ = player − guard, then $\mathbf{f}\cdot\mathbf{rel}$, then the verdict.

player at $(2,9)$: $\mathbf{rel}=($ _____, _____ $)$, dot = _____, seen? ______

player at $(9,3)$: $\mathbf{rel}=($ _____, _____ $)$, dot = _____, seen? ______

**E3.** Same guard, same two players, but now the guard looks *right*: $\mathbf{f}=(1,0)$. Redo both verdicts. What flipped?

**E4.** Narrow the vision to a 45° cone: seen iff $\mathbf{f}\cdot\widehat{\mathbf{rel}}>0.7$, with $\mathbf{f}=(1,0)$. Decide for $\mathbf{rel}=(4,3)$, for $\mathbf{rel}=(3,4)$, and for $\mathbf{rel}=(40,30)$. Why must $\mathbf{rel}$ be made one step long first?

## Part F • Right Angles and Sides

COMPUTE ≈ 5 min

**F1.** The quarter turn of $(5,2)$ is $(-y,x)=($ _____, _____ $)$. Check with a dot product: ____________________ $=0$.

**F2.** Tick every vector perpendicular to $(5,2)$: ☐ $(2,-5)$ ☐ $(-2,5)$ ☐ $(5,-2)$ ☐ $(2,5)$ ☐ $(-4,10)$

**F3.** The 2D cross product $\mathbf{a}\times\mathbf{b}=a_xb_y-a_yb_x$ answers "which side?". With $\mathbf{a}=(5,2)$:
$\mathbf{a}\times(1,4)=$ ______, so $(1,4)$ is on the ________ of $\mathbf{a}$. $\mathbf{a}\times(4,-3)=$ ______, so $(4,-3)$ is on the ________. $(2,3)\times(4,6)=$ ______, meaning ____________.

**F4.** School says two lines are perpendicular when their slopes multiply to $-1$. Try it on $(0,4)$ and $(3,0)$. Then try the dot product. Which test survives?

## Part G • Shadows and Projection

COMPUTE ≈ 7 min

Recipe: $\mathrm{proj}_{\mathbf{b}}(\mathbf{a})=\dfrac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\,\mathbf{b}$. When $\mathbf{b}$ is already one step long, that is just $(\mathbf{a}\cdot\hat{\mathbf{b}})\,\hat{\mathbf{b}}$.

**G1.** $\mathbf{a}=(5,3)$ onto $\hat{\mathbf{b}}=(1,0)$. Shadow length: ______. Shadow as a vector: $($ _____, _____ $)$.

**G2.** $\mathbf{a}=(5,3)$ onto $\mathbf{b}=(0,2)$ — not one step long. $\mathbf{a}\cdot\mathbf{b}=$ _____, $\mathbf{b}\cdot\mathbf{b}=$ _____, ratio _____, projection $($ _____, _____ $)$. Would shrinking $\mathbf{b}$ to one step first give the same answer?

**G3.** A slanted one: $\mathbf{a}=(7,1)$ onto $\mathbf{b}=(3,4)$. Projection: $($ _____, _____ $)$. The "across" part $\mathbf{a}-\text{proj}$: $($ _____, _____ $)$. Check that across is perpendicular to $\mathbf{b}$: ________________.

**G4.** Now the angle between those two, $\mathbf{a}=(7,1)$ and $\mathbf{b}=(3,4)$: $\cos\theta=\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}=$ ________________ $\approx$ ______, so $\theta\approx$ ______.

**G5.** $\mathbf{a}=(2,4)$ and $\mathbf{b}=(4,8)$ point the same way. Compute the ratio above exactly. What could a computer report instead, and what does `acos` do about it? Name the one-line cure.

## Part H • Bounce

COMPUTE ≈ 6 min

Reflection off a wall with one-step-long outward normal $\mathbf{n}$: $\mathbf{r} = \mathbf{d} - 2\,(\mathbf{d} \cdot \mathbf{n})\,\mathbf{n}$. Draw each one — the picture tells you which component should flip.

**H1.** Floor, $\mathbf{n} = (0, 1)$; incoming $\mathbf{d} = (4, -2)$. $\mathbf{d} \cdot \mathbf{n} =$ _____; $\mathbf{r} =$ ( _____, _____ ).

**H2.** Left wall, $\mathbf{n} = (1, 0)$; incoming $\mathbf{d} = (-3, -5)$. $\mathbf{r} =$ ( _____, _____ ). Which component flipped, which stayed?

**H3.** Ceiling, $\mathbf{n} = (0, -1)$; incoming $\mathbf{d} = (2, 3)$. $\mathbf{r} =$ ( _____, _____ ).

**H4.** A slanted wall, $\mathbf{n} = (0.6, 0.8)$; the ball drops straight down, $\mathbf{d} = (0, -5)$. $\mathbf{r} =$ ( _____, _____ ). Verify $|\mathbf{r}| = |\mathbf{d}|$ — a bounce must not change speed.

**H5.** Someone passes the floor's normal as $\mathbf{n} = (0, 2)$ instead of $(0, 1)$, with $\mathbf{d} = (4, -2)$ as in H1. What comes out, and what went wrong?

## Part I • Find the Bug

FIX ≈ 6 min

Each line is *almost* right. Name the bug and give the fix — test your fix on the numbers suggested.

**I1.** Symptom: the guard sees players standing *behind* it and misses the ones in front.

```
rel = guard_pos - player_pos
facing = fwd.dot(rel) > 0
```

Bug & fix: ________________________________________

**I2.** Symptom: normalizing $(6, 8)$ returns $(0.06, 0.08)$.

```
len_sq = x * x + y * y
unit = (x / len_sq, y / len_sq)
```

Bug & fix: ________________________________________

**I3.** Symptom: "perpendicular" arrows come out at odd angles; the arrowheads on a thick line lean.

```
perp = (y, x)
```

Bug & fix: ________________________________________

**I4.** Symptom: the angle function crashes for two vectors pointing the same way, and for a zero-length input.

```
t = a.dot(b) / (a.length() * b.length())
return math.acos(t)
```

Bug & fix: ________________________________________

**I5.** Symptom: the 45° cone test says "inside" for a player far away and almost sideways, at $\mathbf{rel} = (1, 30)$.

```
inside = fwd.dot(rel) > 0.7
```

Bug & fix: ________________________________________

## Part J • One Sentence Each

EXPLAIN ≈ 5 min

**J1.** Why is it safe to compare *squared* lengths when asking "which is longer?"

**J2.** Why do we say a vector "has no home"?

**J3.** Why does the dot product beat measuring angles for "is the guard facing the player?"

**J4.** On screen, $y$ grows *down*. Do any of today's formulas change? What is the outward normal of a floor along the *bottom* edge of the screen?

**J5.** In one sentence: what question does the cross product answer that the dot product cannot?

## Part K • Bonus: The Closest Point on a Segment ★

DESIGN ≈ 10 min

For the brave. A wall is the segment from $A = (1, 1)$ to $B = (9, 5)$. A ball sits at $P$. We want the point $C$ on the segment closest to $P$ — the first step of every ball–wall collision test. The recipe is projection plus one clamp.

**K1.** Write $\mathbf{ab} = B - A$ and $\mathbf{ap} = P - A$ for $P = (3, 7)$. Then compute $t = \dfrac{\mathbf{ap} \cdot \mathbf{ab}}{\mathbf{ab} \cdot \mathbf{ab}}$. What does $t$ measure, in words?

**K2.** The closest point is $C = A + t\,\mathbf{ab}$. Compute it, the distance $|P - C|$, and check that $P - C$ is perpendicular to the wall.

**K3.** Now $P = (12, 9)$. Compute $t$. What is wrong with $A + t\,\mathbf{ab}$ as "the closest point on the segment"? Fix it with one clamp and give the corrected $C$.

**K4.** A ball of radius $r$ at $P$ touches the wall exactly when __________________. Say why the test should compare *squared* quantities.

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**Stuck on more than three items?** Re-read the Lecture 4 slides — every answer above comes straight from them. The one-page cheat sheet near the end is the place to start.

## Answer Key

Check your work *after* attempting everything. If an answer surprises you, that item is worth re-reading in the slides.

**A1.** 4 right, 1 down. It remembers its **direction** and its **length**; the same arrow drawn anywhere on the page is the same vector.

**A2.** location; trip. $Q - P$: destination minus start.

**A3.** $\sqrt{x^2 + y^2}$ (Pythagoras). Length 1. Divide by its own length: $\hat{\mathbf{v}} = \mathbf{v}/|\mathbf{v}|$.

**A4.** $a_x b_x + a_y b_y$. A single **number** comes out, not a vector.

**A5.** roughly the same way; roughly opposite ways; perpendicular (no shadow at all).

**A6.** the shadow that $\mathbf{a}$ casts on $\hat{\mathbf{b}}$'s line, with the sun straight overhead — positive along $\hat{\mathbf{b}}$, negative against it.

**B1.** $(5, 3)$: "5 right, 3 up". Start at $P$, walk it, and you land on $Q$.

**B2.** The first two only. Both are "5 right, 3 up" from different starting points — a vector has no home. The third is $(-5, -3)$, the reverse trip. The fourth is $(5, 2)$: right length in $x$, wrong in $y$.

**B3.** $(6, 4)$. Componentwise: $2 + 4 = 6$ right, $5 - 1 = 4$ up. The sum is the shortcut for the two-leg walk.

**B4.** $(6, -3)$: same way, three times as far. $(-2, -5)$: same length, reversed. $(4, 3)$: same way, half as far.

**B5.** vector (the trip between them); point (go there, arrive); vector (two trips chained); nonsense — adding two *locations* means nothing, even though the code would happily run.

**C1.** $\mathtt{dx} = 9 - 1 = 8$, $\mathtt{dy} = 6 - 2 = 4$. Destination minus start.

**C2.** Moves $(2, 1)$, a quarter of $(8, 4)$. Lands at $(3, 3)$.

**C3.** $\mathtt{dx} = 6$, $\mathtt{dy} = 3$. Moves $(1.5, 0.75)$. Lands at $(4.5, 3.75)$.

**C4.** Slowing down: $(2, 1)$ is longer than $(1.5, 0.75)$, and every nudge is a quarter of *what is left*, so the leftover shrinks but never reaches zero. For a steady speed, use the one-step-long direction $(\mathtt{dx}, \mathtt{dy})$/length times a fixed speed.

**D1.** 10, 13, 17. Note the minus sign vanishes in the square: length never cares about direction.

**D2.** $(0.6, 0.8)$. Check: $\sqrt{0.36 + 0.64} = \sqrt{1} = 1$. Same direction, exactly one step long.

**D3.** Neither — $49 + 1 = 50$ and $25 + 25 = 50$. Equal squared lengths mean equal lengths; the square root changes nothing about the order, so skip it.

**D4.** Trip $(6, 8)$, length 10, direction $(0.6, 0.8)$, nudge $(3, 4)$, new position $(5, 6)$. Two nudges: $10/5$. Unlike Part C, this one really arrives.

**D5.** The trip is $(0, 0)$, its length is 0, and "divide by the length" divides by zero. No direction exists, so we *define* the answer as $(0, 0)$. Guard with a tiny `EPSILON` ("length smaller than this counts as zero"), because float arithmetic rarely lands on exactly 0.0.

**E1.** $6 - 5 = 1 > 0$: same way (only just — nearly a right angle). $-4 + 4 = 0$: perpendicular. $-6 - 5 = -11 < 0$: opposite. $0 + 0 = 0$: perpendicular — one lies on the $x$-axis, the other on the $y$-axis.

**E2.** $(2, 9)$: $\mathbf{rel} = (-2, 5)$, dot $= 0 + 5 = 5 > 0$, **seen** — the player is above the guard, which is where "up" points. $(9, 3)$: $\mathbf{rel} = (5, -1)$, dot $= -1 < 0$, **not seen** — slightly below.

**E3.** $(-2, 5) \cdot (1, 0) = -2 < 0$: not seen. $(5, -1) \cdot (1, 0) = 5 > 0$: seen. Both verdicts flipped — **rel** did not change, only which way the guard faces.

**E4.** $(4, 3)$ has length 5, so $\widehat{\mathbf{rel}} = (0.8, 0.6)$ and the dot is $0.8 > 0.7$: inside. $(3, 4)$ gives $(0.6, 0.8)$, dot $0.6 < 0.7$: outside. $(40, 30)$ normalizes to the same $(0.8, 0.6)$: inside. Without normalizing, $(40, 30) \cdot (1, 0) = 40$ passes any threshold just by being far away — distance would leak into a question about direction.

**F1.** $(-2, 5)$. Check: $5 \cdot (-2) + 2 \cdot 5 = -10 + 10 = 0$. Perpendicular, no slopes needed.

**F2.** $(2, -5)$: $10 - 10 = 0$ yes. $(-2, 5)$: yes. $(5, -2)$: $25 - 4 = 21$ no. $(2, 5)$: $10 + 10 = 20$ no. $(-4, 10)$: $-20 + 20 = 0$ yes — it is $2 \cdot (-2, 5)$; any multiple of a perpendicular is perpendicular.

**F3.** $5 \cdot 4 - 2 \cdot 1 = 18 > 0$: left. $5 \cdot (-3) - 2 \cdot 4 = -23 < 0$: right. $2 \cdot 6 - 3 \cdot 4 = 0$: parallel — $(4, 6)$ is just $2 \cdot (2, 3)$.

**F4.** $(0, 4)$ is vertical: its slope is $4/0$, undefined — the slope test cannot even start. The dot product is $0 \cdot 3 + 4 \cdot 0 = 0$: perpendicular, no special case. Graphics is full of vertical lines, which is why we use the dot.

**G1.** Shadow length $5 \cdot 1 + 3 \cdot 0 = 5$. As a vector: $5 \cdot (1, 0) = (5, 0)$ — keep the "along $x$" part, drop the rest.

**G2.** $\mathbf{a} \cdot \mathbf{b} = 6$, $\mathbf{b} \cdot \mathbf{b} = 4$, ratio 1.5, projection $1.5 \cdot (0, 2) = (0, 3)$. Yes: $\hat{\mathbf{b}} = (0, 1)$, shadow 3, vector $(0, 3)$. The division by $\mathbf{b} \cdot \mathbf{b}$ does the shrinking for you.

**G3.** $\mathbf{a} \cdot \mathbf{b} = 21 + 4 = 25$, $\mathbf{b} \cdot \mathbf{b} = 25$, ratio 1, so the projection is $(3, 4)$ itself. Across $= (7, 1) - (3, 4) = (4, -3)$. Check: $(4, -3) \cdot (3, 4) = 12 - 12 = 0$. Every vector splits into along + across.

**G4.** $25/(\sqrt{50} \cdot 5) = 25/35.36 \approx 0.707$, so $\theta \approx 45^\circ$ — which the previous item already showed: along $(3, 4)$ and across $(4, -3)$ have equal length 5, a $45^\circ$ split.

**G5.** $\mathbf{a} \cdot \mathbf{b} = 8 + 32 = 40$; $|\mathbf{a}||\mathbf{b}| = \sqrt{20}\sqrt{80} = \sqrt{1600} = 40$; ratio exactly 1, angle $0^\circ$. Float rounding can return 1.0000000000000002, and `acos` refuses anything outside $[-1, 1]$ with a domain error. Cure: clamp the ratio into $[-1, 1]$ before calling `acos`.

**H1.** $\mathbf{d} \cdot \mathbf{n} = -2$. $\mathbf{r} = (4, -2) - 2(-2)(0, 1) = (4, -2) + (0, 4) = (4, 2)$. Still 4 to the right, now 2 *up*.

**H2.** $\mathbf{d} \cdot \mathbf{n} = -3$; $\mathbf{r} = (-3, -5) + 6(1, 0) = (3, -5)$. The $x$ component flipped (into the wall becomes out of it); the $y$ component, sliding along the wall, is untouched.

**H3.** $\mathbf{d} \cdot \mathbf{n} = -3$; $\mathbf{r} = (2, 3) - 2(-3)(0, -1) = (2, 3) + (0, -6) = (2, -3)$. Watch the double negative: $-2 \cdot (-3) = +6$, times $(0, -1)$ points down.

**H4.** $\mathbf{d} \cdot \mathbf{n} = -4$; $\mathbf{r} = (0, -5) + 8 \cdot (0.6, 0.8) = (4.8, 1.4)$. Length: $\sqrt{23.04 + 1.96} = \sqrt{25} = 5 = |\mathbf{d}|$. Speed kept; only the direction changed.

**H5.** $\mathbf{d} \cdot \mathbf{n} = -4$; $\mathbf{r} = (4, -2) - 2(-4)(0, 2) = (4, -2) + (0, 16) = (4, 14)$ — a ball that leaves seven times faster than it arrived. The formula assumes $\mathbf{n}$ is one step long; normalize $\mathbf{n}$ first (or use the projection recipe with $\mathbf{n} \cdot \mathbf{n}$ in the denominator).

**I1.** The subtraction is backwards: it builds the trip from the player *to* the guard, which reverses the sign of every dot product. Fix: `rel = player_pos - guard_pos` — destination minus start.

**I2.** It divides by the *squared* length, 100, instead of the length, 10. Fix: take the square root first, `length = math.sqrt(len_sq)`, then divide by `length`. Result $(0.6, 0.8)$.

**I3.** Swapped but not negated. For $(5, 2)$ it gives $(2, 5)$, and $(5, 2) \cdot (2, 5) = 20 \neq 0$. Fix: negate one component, `perp = (-y, x)`, the quarter turn.

**I4.** Two bugs. Float rounding can push `t` a hair past 1 and `acos` raises a domain error — clamp with `t = max(-1.0, min(1.0, t))`. And a zero-length input makes the denominator 0 — if either length is below `EPSILON`, return `0.0` (no angle exists, so define one).

**I5.** `rel` was never made one step long: $(1, 0) \cdot (1, 30) = 1 > 0.7$ passes on distance alone. Fix: `fwd.dot(rel.normalized()) > 0.7`; then the value is $1/\sqrt{901} \approx 0.03$, far outside the cone.

**J1.** Squaring keeps the order of non-negative numbers, so the bigger squared length belongs to the bigger length — the square root would change the numbers, not the verdict.

**J2.** "5 right, 3 up" is the same instruction from any starting point; a vector stores direction and length only, and the start point belongs to whoever uses it.

**J3.** It needs two multiplies and one add, no trigonometry, and it never wraps around — angles jump from 359° to 0°, a sign does not.

**J4.** No formula changes — they never assumed which way $y$ points. The floor's outward normal points *up* on the screen, which in screen coordinates is $(0, -1)$. Only your drawings appear mirrored.

**J5.** Which *side* of $\mathbf{a}$ the vector $\mathbf{b}$ lies on (left or right, or neither if parallel) — the dot only says "same way or not".

**K1.** $\mathbf{ab} = (8, 4)$, $\mathbf{ap} = (2, 6)$, $t = (16 + 24)/(64 + 16) = 40/80 = 0.5$. It is the shadow of $\mathbf{ap}$ on the wall, measured as a *fraction* of the wall's length: 0 at $A$, 1 at $B$.

**K2.** $C = (1, 1) + 0.5 \cdot (8, 4) = (5, 3)$. $P - C = (-2, 4)$, distance $\sqrt{4 + 16} = \sqrt{20} \approx 4.47$. Check: $(-2, 4) \cdot (8, 4) = -16 + 16 = 0$ — the shortest path to a line always meets it at a right angle.

**K3.** $\mathbf{ap} = (11, 8)$, $t = (88 + 32)/80 = 1.5$. $A + 1.5\,\mathbf{ab} = (13, 7)$ lies on the wall's *line* but past $B$, where there is no wall. Clamp $t$ into $[0, 1]$: $t = 1$, so $C = B = (9, 5)$. The nearest point of a segment to something beyond its end is the end itself.

**K4.** When $|P - C| \leq r$, with $C$ the clamped closest point. Compare $(P - C) \cdot (P - C) \leq r^2$ instead — same verdict, no square root, and this runs for every ball against every wall every frame of an animation.